Course Goals: Math 461, Geometry

0. Preliminaries

Students will be able to:

- Describe the central role that equivalence relations play in mathematics, citing several common examples of equivalence relations, including set equality, congruence, similarity, and isometric.

I. Axiomatic Systems

Students will be able to:

- Take a simple set of axioms, develop a model for the axiomatic system, make conjectures, find counterexamples, and prove elementary theorems for that axiomatic system.
- Demonstrate knowledge of the axioms of finite projective and affine geometries, be able to prove standard theorems of that system and provide models for that system and counterexamples to various statements.

II. History

Students will be able to:

- Demonstrate familiarity with the history of parallel postulate and the development of non-Euclidean geometries.
- Describe the contributions of various geometers, such as Euclid, Sacharri, Gauss, Borchelli, Lobeschevski, Riemann and Klein.

III. Euclidean Plane, E^2

Students will be able to:

- Demonstrate familiarity with the Euclidean axioms, and the ability to prove elementary theorems of Euclidean geometry.
- Analyze situations, solve geometric and/or trigonometric problems using analytic, dynamic and/or synthetic means and provide .
- Determine whether or not any given transformation of the Euclidean plane E^2 is an isometry and to show that the function preserves distance.

- Classify the isometries of the Euclidean plane E^2.
- Prove theorems involving isometries of the plane such as the three reflection theorem.
- Solve geometric problems using isometries.
- Explain why various triangle theorems, such as SSS, SAS, ASA, work on the Euclidean plane E^2.

IV. Dynamic Geometry

Students will be able to:

- Use *Geometer's Sketchpad* to make conjectures and test hypotheses about Euclidean and hyperbolic geometry.
- Use *Geometer's Sketchpad* to generate various curves, objects and shapes.
- Use *Geometer's Sketchpad* to analyze and solve various geometric and/or trigonometric problems.
- Use *Geometer's Sketchpad* to create Escher-like tesselations and tilings of the plane.

V. Covering Spaces, π: XY

Students will be able to:

- Demonstrate familiarity with the concept, definition and role of covering spaces in the study of Euclidean surfaces, including simple examples such as e^{pit}: E^1 S^1 =$\{(x,y)|x^2 + y^2 = 1\}$ = E^1/**Z C** @ E^2.
- Demonstrate familiarity with the concept, definition and role of covering isometry groups and quotient spaces.
- Determine conditions of when an isometry of the space X will induce isometries of the space Y using knowledge of the covering isometry groups and quotient spaces of the covering space π: XY.
- Determine which isometries on the Euclidean plane E^2 will induce isometries on a given Euclidean surface.

VI. Cone

Students will be able to:

- Describe what is straight on the cone.
- Describe the properties of geodesics on the cone.

VII. Euclidean Surfaces

Students will be able to:

- Classify all of the closed Euclidean surfaces.
- Classify the geodesics of each Euclidean surface.
- Describe all of the isometries of each Euclidean surface and justify that classification.
- Demonstrate familiarity with the infinite Cylinder, the twisted cylinder (infinite Mobius Strip), the torus (both in E^3 with induced metric and in E^4 with Euclidean metric) and the Klein bottle (in E^4 with Euclidean metric), being able to indicate the covering group which generated each space as a quotient space of the Euclidean plane E^2.

VIII. Spherical Geometry, S^2

Students will be able to:

- Describe what is straight on the sphere S^2.
- Describe the properties of geodesics on the sphere S^2.
- Determine which triangle theorems, such as SSS, SAS, ASA, work on the sphere S^2.
- Classify the isometries of the sphere S^2.
- Express the area of a triangle or polygon on the sphere S^2 in terms of sum of the interior or exterior angles of the triangle, polygon, respectively (Gauss-Bonnett Theorem for spheres).
- Prove theorems involving isometries of the sphere.
- Demonstrate familiarity with various mappings of the sphere to the plane including stereographic, mercator, cylindrical projections.

IX. Tesselations and Tilings

Students will be able to:

- Classify all of the regular tilings of the Euclidean plane E^2.
- Classify all of the semi-regular tilings of the Euclidean plane E^2.
- Classify all of the regular tilings of the sphere S^2.
- Classify all of the semi-regular tilings of the sphere S^2.
- Analyze the underlying isometries used to generate a tesselation of either the Euclidean plane E^2 or sphere S^2.

X. Projective Plane, P^2

Students will be able to:

- Demonstrate familiarity with the relationship between perspective drawing and the projective plane.
- Demonstrate familiarity with the axioms of the projective plane and how they compare with the Eucidean axioms, and the ability to prove elementary theorems of projective geometry.
- Determine the relationship between sum of the angles of a triangle in the projective plane and the parallel postulate.

XI. Hyperbolic Plane, H^2

Students will be able to:

- Demonstrate familiarity with various models of the hyperbolic plane, the pseudo sphere, the Poincare, Klein and half-plane models and the relationship between them.
- Using the half-plane model of the hyperbolic plane, determine the distance between any two points in the plane.
- Using the half-plane model of the hyperbolic plane, determine the area of a triangle in the hyperbolic plane.
- Determine the relationship between sum of the angles of a triangle in the hyperbolic plane and the parallel postulate

XII. Differential Geometry

Students will be able to:

- Calculate the curvature, torsion of a parameterized path in either the plane or in space.
- Demonstrate familiarity with the Frenet frame TBN.
- Calculate the first and second fundamental forms of a point on a given surface.
- Calculate the Gaussian curvature at any point on a given surface of revolution.

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Revised: June 07, 2010

For more information please contact: jewell@edgewood.edu