Course Goals: Math 331, Differential Equations

Prerequisite Knowledge from Calculus

I. Differentiation

Students should already be able to demonstrate:

- A solid understanding of the derivative,
- The ability to apply the rules of differentiation to an arbitrary function consisting of basic function components to find the derivative of any arbitrary function,
- The ability to interpret the derivative in either graphical, tabular or algebraic form to determine the concavity, increasing, decreasing, local extrema and critical points of the original function,
- The ability to find critical points of a function of one variable and determine both local and global extrema of the function,
- The ability to take a real-world extrema problem and to describe a function, possibly of more than one variable, to be maximized or minimized, subject to possible constraint equations, and by means of substitution using the constraint equations, reduce the function to a function of just one independent variable.

II. Integration

Students should already be able to demonstrate:

- A solid understanding of both the definite and indefinite Riemann integral,
- The ability to use substitutions and integration by parts to evaluate various integrals and/or find the antiderivatives of an arbitrary function consisting of basic function components,
- A recognition that, unlike with differentiation, some functions do not have antiderivatives which can be expressed in a finite number of components using basic functions,
- The ability to take a real-world problem relating one variable as a product of two variables where one is a function of the other, by slicing the relevant object into small pieces in such a way that the variables are nearly constant on each piece, use the formula relating the quantities on each piece to and obtain a definite integral to find the quantity over the entire object.

III. Relating Derivatives and Integrals

Students should already be able to demonstrate:

- A solid understanding of the first and second forms of the fundamental theorem of calculus,

- Given the graph of a function, the ability to draw the graph of both its derivative and its antiderivative functions using information about the relationship between the derivative of a function and the functions concavity, increasing, decreasing, local extrema and inflection points, and also using the fundamental theorem of calculus to approximate the change in height of the antiderivative function.
- The ability to solve differential equations of the form y = f(x) by evaluating the indefinite integral, $y = \int f(x)dx$.

IV.Approximation and Series

Students should already be able to demonstrate:

- The ability to explain why the n-degree Taylor polynomial is the best fitting n-degree polynomial approximation to a given function near a given point,
- The ability to find the Taylor polynomial or series for simple functions and to use known Taylor series to find new Taylor series by means of substitutions, integration or differentiation, term by term,
- The ability to determine the nth term of a given series and to utilize summation notation in addition to expanding summations out term by term,
- The ability to determine intervals of convergence.

Goals for Differential Equations

I. General

Students should be able to demonstrate:

- The ability to classify differential equations as ordinary or partial, linear or nonlinear, homogeneous or nonhomogeneous, and its order, boundary values and/or initial conditions,
- An understanding of the existence and uniqueness theorems for first and second order ordinary differential equations,
- An ability to apply integrating factors to simplify differential equations,
- An ability to use linear operators and recognize that differential operators are linear operators,
- An understanding of linear independence of functions,
- An ability to use the Wronskian in determining solutions to differential equations,
- An ability to apply basic matrix and vector algebra and recognize the role it plays in solving systems of linear equations.

II. First Order Ordinary Differential Equations

Students should be able to demonstrate:

- The ability to solve differential equations of the form y = f(x) by evaluating the indefinite integral, $y = \int f(x)dx$,
- The ability to provide solution equations relating the x and y variables for separable differential equations of the form, $dy/dx = f(x)g(y)$,

- The ability to determine whether or not a differential equation is either exact or separable and use their associated techniques to solve the differential equation,
- The ability to find the solution to a linear homogeneous ordinary differential equations with constant coefficients, $y + ay = 0$,
- The ability to find the solution to a linear homogeneous ordinary differential, $y + p(x)y = 0$,
- The ability to find the solution to a linear nonhomogeneous ordinary differential, $y + p(x)y = g(x)$,
- Given a first order differential equation, $y = f(x,y)$, the ability to sketch the slope fields and solutions curves for given initial conditions,
- Given a slope field for a given differential equation, guess a possible algebraic solution and verify that it satisfies the differential equation,
- The ability to take a real-world problem that can be expressed as the solution to a first order differential equation, articulate that governing differential equation and provide its solution when possible.

III. Second Order Ordinary Differential Equations

Students should be able to demonstrate:

- The ability to solve differential equations of the form $y = f(x)$ by reducing it to the differential equation, $y = \int f(x)dx$, and solving that differential equation,
- The ability to reduce the order of a second order homogeneous linear differential equation if you know one solution to the equation,
- The ability to find the homogeneous solution to a linear homogeneous ordinary differential equations with constant coefficients, $ay + by + cy = 0$, and the role that the characteristic equation, $ar^2 + br + c = 0$, in finding that solution,
- The ability to use the method of undetermined coefficients to find the solution to a linear nonhomogeneous ordinary differential with constant coefficients, $ay + by + cy = g(x)$,
- The ability to find the solution to a linear homogeneous ordinary differential, $y + p(x)y + q(x)y = 0$, using integrating factors,
- The ability to use the method of variation of parameters to find the solution to a linear nonhomogeneous ordinary differential, $y + p(x)y + q(x)y = g(x)$,
- The ability to take a real-world problem that an be expressed as the solution to a second order linear differential equation with constant coefficients, articulate that governing differential equation and provide its solution when possible.

IV. Power Series Solutions

Students should be able to demonstrate:

- The ability to shift indices in summations and perform simple algebraic manipulations involving summations, including factoring terms out of summations and distributing terms into summations,
- The ability to solve second order linear homogeneous differential equations of the form, $P(x)y + Q(x)y + R(x)y = 0$ using power series,
- The ability to recognize common Taylor series solutions to some second order linear homogeneous differential equations when using power series.

V. Higher Order Linear Ordinary Differential Equations

Students should be able to demonstrate:

- The ability to find the homogeneous solution to a higher order linear homogeneous ordinary differential equations with constant coefficients, and the role that the characteristic equation in finding that solution,
- The ability to use the method of undetermined coefficients to find the solution to a higher order linear nonhomogeneous ordinary differential with constant coefficients.

VI. Systems of First Order Linear Ordinary Differential Equations

Students should be able to demonstrate:

- The ability to formulate a system of first order, linear, homogeneous, ordinary differential equations as a matrix equation, $\mathbf{x} = A\mathbf{x}$, with basic solutions of the form $\mathbf{x} = \mathbf{u}e^{\lambda t}$.
- The ability to set up and solve the *eigenproblem* $A\mathbf{x} = \lambda\mathbf{x}$, to determine the eigenvalues λ and their associated eigenvectors $\mathbf{u}$.
- The ability to determine the general solution of the system of first order, linear, homogeneous, ordinary differential equations, $\mathbf{x} = A\mathbf{x}$, using the eigenvalues, real, distinct or repeated, or complex, and their associated eigenvectors.

Approved April 7, 2002

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Revised: June 07, 2010

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