Course Goals: Math 231, First Semester Calculus

Catalogue description: An introduction to differential and integral calculus and plane analytic geometry. Derivatives are developed for algebraic functions, logarithmic and exponential functions for various bases, and trigonometric functions and their inverses. The Riemann integral and the first form of the Fundamental Theorem of Calculus are introduced. Definite integrals and anti-derivatives are developed for basic algebraic, exponential, logarithmic, and trigonometric functions. Applications of key concepts are integrated throughout the course. *Prerequisites*: Successful completion of Math 114B with a grade of C or above, or placement.

Course Goals: Students should be able to demonstrate the following capacities in each area:

Pre-requisite skills

- A basic understanding of the fundamental functions: polynomial, trigonometric, exponential and logarithmic, rational and power functions,
- A basic understanding of intercepts, asymptotes, periodicity, concavity, increasing and decreasing,
- The ability to solve equations for independent variables,
- Familiarity with the composition of functions, inverses and the properties $f(f^{-1}(x)) = f^{-1}(f(x)) = x$, and change of variables, also called substitutions,
- Familiarity with basic transformations: horizontal and vertical shifts, reflections and scaling,
- An intuitive understanding of continuity and notion of limits,
- The ability to calculate limits.

II. Concept of Differentiation

- A solid understanding of the definition of the derivative,
- The ability to approximate the derivative of a function at a point and as a function given in either tabular, graphical or algebraic form,
- The ability to compute the derivative algebraically using the definition,
- The ability to apply the derivative as the slope of the tangent line or as an instantaneous rate of change in applications, the most common of which is velocity as the slope of the distance versus time graph,
- The ability to determine information about the concavity, increasing and decreasing, local extrema, and inflection points of a function given its derivative in either tabular, graphical or algebraic form.

III. Short-cuts to Derivatives

- The ability to derive the derivatives of the basic functions using the definition of the derivative,
- The ability to derive the basic rules of differentiation (sum, differences, product) using the definition of the derivative,
- The ability to apply the chain rule and properties of inverse functions to derive the derivatives of the inverses of basic functions,
- The ability to apply the rules of differentiation to an arbitrary function consisting of basic function components to find the derivative of any arbitrary function.

IV. Application of Derivatives

- The ability to interpret the derivative in either graphical, tabular or algebraic form to determine the concavity, increasing, decreasing, local extrema and critical points of the original function,
- The ability to find critical points of a function of one variable and determine both local and global extrema of the function,
- The ability to justify why a certain critical point of a function is a local extremum using information about either the first derivative or the second derivative if the derivative of the critical point is zero,
- The ability to take a real-world extrema problem and to describe a function, possibly of more than one variable, to be maximized or minimized, subject to possible constraint equations, and by means of substitution using the constraint equations, reduce the function to a function of just one independent variable.

V. Concept of Definite Integral

- A solid understanding of the definition of the Riemann integral,
- The ability to approximate the definite integral of a function given in either tabular, graphical or algebraic form,
- The ability to set up Riemann sums and compute either the left hand or right hand sums using the definition of the Riemann integral using uniform widths Δx, including taking the limit as the uniform width Δx goes to 0,
- The ability to take a sum and interpret it as a Riemann sum approximating the integral of some function over some interval,
- The ability to apply the definite integral as the signed area under the curve over an interval in applications, the most common of which is the change in distance as the signed area in the velocity versus time graph.

VI. Relating Derivatives and Integrals

- A solid understanding of the first form of the fundamental theorem of calculus, being able to both state and apply it,
- Given the graph of a function, the ability to draw the graph of both its derivative and its antiderivative functions using information about the relationship between the derivative of a function and the function's concavity, increasing, decreasing, local extrema and inflection points, and also using the fundamental theorem of calculus to approximate the change in height of the antiderivative function.

Students must pass a **Math 231 – Differentiation Benchmark Exam** in order to receive a grade of C or better in this course. The benchmark consists of 10 differentiation problems; students will have 30 minutes to complete the exam and will not be permitted to use a calculator. Passing requires 8 or more correct answers. The Math 231 – Benchmark may be taken repeatedly till passing is achieved during the semester in which the student is registered for Math 231.

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