Course Goals: Math 442, Abstract Algebra II

Catalog description: This course continues the study of abstract algebra and is focused mainly on groups, with some exploration of rings and fields as time allows. Group theoretic topics include subgroups, normal subgroups and quotient groups, and some counting principles. A wide variety of examples will be explored.

Learning Objectives:

I. Algebra fundamentals
Students will be able to:

- Define equivalence relation and equivalence class and determine, with complete justification, whether or not a given relation is an equivalence relation and, if so, identify equivalence classes.
- State the Well-Ordering Principle of the positive integers and use it in a proof.
- State and use the First and Second Form of Mathematical Induction and it in a proof.
- Define function, one-to-one function, and onto-function; and identify examples and non-examples of each.
- Define left-inverse, right-inverse and inverse of a function; and identify examples and non-examples of each, and prove the equivalence of one-to-one and existence of a left-inverse, and the equivalence of onto with existence of a right inverse.

II. Groups
Students will be able to:

- Demonstrate familiarity with the definition of a group and be able to test a set with binary operation to determine if it is a group.
- Construct a Cayley table for a group.
- Demonstrate familiarity with the common groups (Z_n, R^*, U(n), GL(2,R), D_n etc)
- Compute the order of a group, the order of a subgroup, and the order of an element.
- Identify subgroups of a given group.
- Identify cyclic groups and apply the fundamental theorem of cyclic groups.
- Demonstrate familiarity with permutation groups and be able to decompose permutations into 2-cycles.
- Define the concepts of homomorphism, isomorphism, and automorphism and check whether a given function defines one of these.
- Prove the common properties of homomorphisms (e.g. that if K is a subgroup of G then $\varphi(K)$ is a subgroup of $\varphi(G)$).
- Prove that Aut(G) is a group and compute Aut(G) for given a given G.
- Define the external direct product and be able to compute the direct product of groups.
- Apply Lagrange's theorem.
- Define normal subgroups and be able to prove that given subgroups are normal.
- State and apply the fundamental theorem of finite Abelian groups.

III. Rings
Students will be able to:

- Give a definition of *ring* and cite a variety of common examples and non-examples (finite and infinite, polynomials, and matrices)

IV. Fields
Students will be able to:

- Give the definition of *field* and cite a variety of common examples and non-examples (characteristic 0 and characteristic *p*, polynomials, matrices)

IV. Applications

Students will be able to:

- Demonstrate familiarity with some of the applications of algebra to other fields, e.g. cryptography.

Approved: 05-08