Course Goals: Math 232, Second-Semester Calculus

This course is a continuation of differential and integral calculus. Topics include integration techniques, improper integrals, applications, differential equations, Taylor polynomials, and infinite series. This course emphasizes the mastery of key concepts and their applications. Prerequisites: Successful completion of MATH 231 with a grade of C or above.

I. Concept of Definite Integral

Students should be able to demonstrate:

- A solid understanding of the definition of the Riemann integral,
- The ability to approximate the definite integral of a function given in either tabular, graphical or algebraic form,
- The ability to set up Riemann sums and compute either the left hand or right hand sums using the definition of the Riemann integral using uniform widths Δx, including taking the limit as the uniform width Δx goes to 0,
- The ability to take a sum and interpret it as a Riemann sum approximating the integral of some function over some interval,
- The ability to apply the definite integral as the signed area under the curve over an interval in applications, the most common of which is the change in distance as the signed area in the velocity versus time graph.

II. Relating Derivatives and Integrals

Students should be able to demonstrate:

- A solid understanding of the first form of the fundamental theorem of calculus, being able to both state and apply it,
- Given the graph of a function, the ability to draw the graph of both its derivative and its antiderivative functions using information about the relationship between the derivative of a function and the function's concavity, increasing, decreasing, local extrema and inflection points, and also using the fundamental theorem of calculus to approximate the change in height of the antiderivative function.

III. Antidifferentiation and Techniques of Integration

Students should be able to demonstrate:

- The ability to use the Fundamental Theorem of Calculus for simple functions 1) to find the antiderivative of the function, 2) to evaluate indefinite integrals of the function, and 3) to evaluate definite integrals of the function,
- Given the graph of a function, the ability to draw the graph of its antiderivative function using information about the relationship between derivatives of functions and the function's concavity, increasing, decreasing, local extrema and inflection points, and using the fundamental theorem of calculus to approximate the change in height of the antiderivative function,

- A solid understanding of the second form of the fundamental theorem of calculus, being able to both state and apply it,
- The ability to derive the basic rules of integration (sum, differences, scalar multiplication) using the Fundamental Theorem of Calculus,
- The ability to derive the various techniques of substitution and integration by parts using the Fundamental Theorem of Calculus and various rules for differentiation,
- The ability to use substitutions and integration by parts to evaluate various integrals and/or find the antiderivatives of an arbitrary function consisting of basic function components,
- A recognition that, unlike with differentiation, some functions do not have antiderivatives which can be expressed in a finite number of components using basic functions,
- The ability to solve differential equations of the form $y' = f(x)$ by evaluating the indefinite integral, $y = \int f(x)dx$.

IV. Applications of Integrals

Students should be able to demonstrate:

- The ability to approximate the definite integral of a function given in either tabular, graphical or algebraic form,
- The ability to set up infinitesimal volumes $\Delta y = A\ \Delta x$ of a region by taking slices of the region perpendicular to a given axis, which we'll call the x-axis, and determine the cross-sectional area A perpendicular to the x-axis as a function of x,
- The ability to find finite volumes of revolution by 1) expressing the radius,r, of revolution as a function of the variable, x, in the direction of the axis of revolution, $r = g(x)$, where $\Delta V = \pi[f(r)]^2\Delta x = \pi[f(g(x))]^2\Delta x$, and 2) expressing the height, h, in the direction of the axis of revolution as a function of the radius, r, measured perpendicularly from the axis of revolution, $h = g(r)$, where $\Delta V = 2\pi rh\Delta r = 2\pi rg(r)\Delta r$,
- For various applications in physics and other fields where one quantity varies continuously as a product of two other quantities, for example $W = Fx$, where F is not necessarily a constant with respect to the variable x, i.e., $F = F(x)$, the ability to slice the object into small pieces in such a way that the variables are nearly constant on each piece, use the formula relating the quantities on each piece to and obtain a definite integral to find the quantity over the entire object, e.g., $W = \int F(x)dx$.

V. Approximation and Series

Students should be able to demonstrate:

- The ability to derive the geometric series by looking at the pattern formed by its partial sums.
- The ability to explain why the n-degree Taylor polynomial is the best fitting n-degree polynomial approximation to a given function near a given point,
- The ability to find the Taylor polynomial or series for simple functions,
- The ability to use known Taylor series to find new Taylor series by means of substitutions, integration or differentiation, term by term,

- The ability to determine the nth term of a given series and to utilize summation notation in addition to expanding summations out term by term,
- The ability to determine intervals of convergence,
- The ability to apply Taylor series to make estimations.

VI. Differential Equations

Students should be able to demonstrate:

- Given a differential equation, the ability to sketch the slope fields and solutions curves for given initial conditions,
- Given a slope field for a given differential equation, the ability to guess a possible algebraic solution and verify that it satisfies the differential equation,
- The ability to solve differential equations of the form $y' = f(x)$ by evaluating the indefinite integral, $y = \int f(x)dx$,
- The ability to solve differential equations of the form $y'' = f(x)$ by reducing it to the differential equation, $y' = \int f(x)dx$, and solving that differential equation,
- The ability to provide solution equations relating the x and y variables for separable differential equations of the form, $dy/dx = f(x)g(y)$,
- The ability to solve first and second order homogeneous linear differential equations with constant coefficients, $ay' + by = 0$, $ay'' + by' + y = 0$,
- The ability to solve first and second order nonhomogeneous linear differential equations with constant coefficients, $ay' + by = f(x)$, $ay'' + by' + y = f(x)$ for functions $f(x)$ involving polynomials, exponentials and sines and cosines.

Students must pass a **Math 232 – Integration Benchmark Exam** in order to earn a grade of C or higher in the course. The benchmark consists of 10 integration problems; students will have 30 minutes to complete the exam and will not be permitted to use a calculator. Passing requires 8 or more correct answers. The Math 232 – Benchmark may be taken repeatedly till passing is achieved during the semester in which the student is registered for Math 232.

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